# Problem Definitions and Evaluation Criteria for CEC 2011 Competition on Testing Evolutionary Algorithms on Real World Optimization Problems

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A comprehensive account of some real world optimization problems have been presented in details. These problems can be used to evaluate the performance of different stochastic optimization algorithm. After all, every optimization algorithm has to be applied to some real world problems. In light of this fact, some real world optimizations problems have been selected and detailed analysis of those problems have given to understand it and the objective functions have been evaluated respected to the constraints (if any) to optimize and get its solution.

### 1. Problem Definitions

In this section some real world problems to evaluate the performance of algorithms are defined.

# 1. Parameter Estimation for Frequency-Modulated (FM) Sound Waves (Problem No.1 in Table 1)

Frequency-Modulated (FM) sound wave synthesis has an important role in several modern music systems and to optimize the parameter of an FM synthesizer is a six dimensional optimization problem where the vector to be optimized is  $X = \{a_1, \omega_1, a_2, \omega_2, a_3, \omega_3\}$  of the sound wave given in eqn. (1). The problem is to generate a sound (1) similar to target sound (2). This problem is a highly complex multimodal one having strong epistasis, with minimum value  $f(\vec{X}_{sol}) = 0$ . This problem has been tackled using Genetic Algorithms (GAs) in [1], [2]. The expressions for the estimated sound and the target sound waves are given as:

$$y(t) = a_1 \cdot \sin\left(\omega_1 \cdot t \cdot \theta + a_2 \cdot \sin\left(\omega_2 \cdot t \cdot \theta + a_3 \cdot \sin\left(\omega_3 \cdot t \cdot \theta\right)\right)\right) \tag{1}$$

$$y_0(t) = (1.0).\sin((5.0)t.\theta - (1.5).\sin((4.8)t.\theta + (2.0).\sin((4.9)t.\theta)))$$
 (2)

respectively where  $\theta = 2\pi/100$  and the parameters are defined in the range  $\begin{bmatrix} -6.4 & 6.35 \end{bmatrix}$ The fitness function is the summation of square errors between the estimated wave (1) and the target wave (2) as follows:

$$f(\vec{X}) = \sum_{t=0}^{100} (y(t) - y_0(t))^2$$
 (3)

### 2. Lennard-Jones Potential Problem (Problem No. 2 in Table 1)

It is a potential energy minimization problem. Lennard-jones potential problem involves the minimization of molecular potential energy associated with pure Lennard-Jones (LJ) cluster [3], [4]. This is a multi-modal optimization problem comprised of an exponential number of local minima [3]. The lattice structure of LJ cluster has an icosahedral core and a combination of surface lattice points. Most of the global minima have structures based upon the Mackay icosahedrons an can be seen in the Cambridge Cluster Database (http://www-wales.ch.cam.ac.uk/CCD.html). An algorithm can be tasted over this function for its capability to conform molecular structure, where the atoms are organized in such a way that the molecule has minimum energy. Lennard-Jones pair potential for *N* atoms, given by the Cartesian coordinates

$$\vec{p}_i = \{\vec{x}_i, \vec{y}_i, \vec{z}_i\}, \quad i = 1, ..., N$$
 (4)

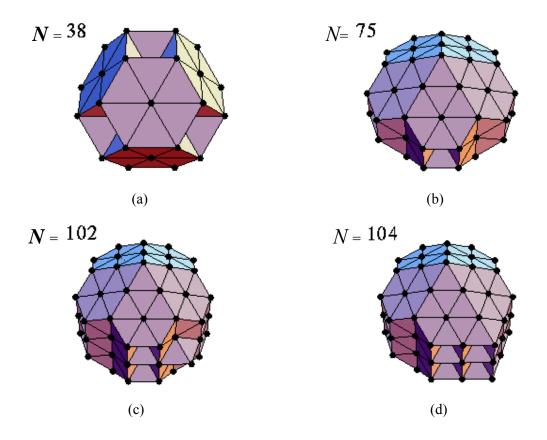
is given as follows:

$$V_N(p) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left( r_{ij}^{-12} - 2.r_{ij}^{-6} \right), \tag{5}$$

where  $r_{ij} = \|\vec{p}_j - \vec{p}_i\|_2$  with gradient

$$\nabla_{j}V_{N}(p) = -12\sum_{i=1,i\neq j}^{N} \left(r_{ij}^{-14} - r_{ij}^{-8}\right) \left(\vec{p}_{j} - \vec{p}_{i}\right), \quad j = 1,...,N$$
(6)

Lennard-Jones potential has minimum value at a particular distance between the two points [5]. The structures of LJ clusters after optimization take following shapes as reported by Cambridge Cluster Database.



**Fig. 1:** (a), (b), (c), and (d) are the shapes for 38, 75, 102 and 104 atom LJ clusters and the optimum potential reported up to now are -173.928427, -402.894866, -569.363652 and -582.086642 respectively pair well depth.

The variation of Lennard-Jones pair potential  $V(r) = r^{-12} - 2 \cdot r^{-6}$  with pair distance r have been plotted in Figure 2. It can be clearly seen that it has a unique minimum value of -1 for a particular value of r (=1). Its value increases very fast when r is decreased from the optimum value and tends to infinity near r=0. This tendency of pair potential curve makes it a hard optimization problem.

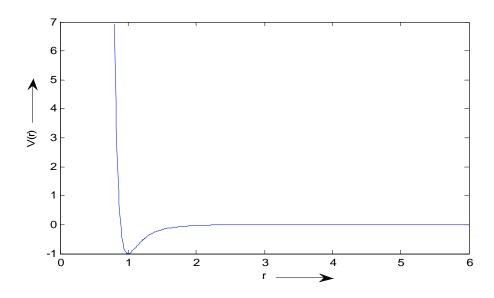


Fig. 2: Variation of pair potential with r

Following [4], we can reduce the dimension of the problem in the following way. We first fix an atom at the origin and choose our second atom to lie on the positive X-axis. The third atom can then be selected to lie in the upper half of the X-axis. Since the position of the first atom is always fixed and the second atom is restricted to the positive X-axis, this gives a minimization problem involving three variables for three atoms. For four atoms, additionally three variables (the Cartesian co-ordinates of the 4-th atom) are required to give a minimization problem in six independent variables. For each further atom, three variables (coordinates of the position of the atom) are added to determine the potential energy of clusters. Let  $\vec{x}$  be the variable of the problem which has three components for three atoms, six components for 4 atoms and so on. The first variable due to the second atom i.e.  $x_1 \in [0,4]$ , then the second and third variables are such that  $x_2 \in [0,4]$  and  $x_3 \in [0,\pi]$ . The coordinates  $x_i$  for any other atom is taken to be bound in the range:

$$\left[ -4 - \frac{1}{4} \left| \frac{i-4}{3} \right|, 4 + \frac{1}{4} \left| \frac{i-4}{3} \right| \right]$$

where  $\lfloor r \rfloor$  is the nearest least integer w.r.t.  $r \in \square$  .

# 3. The Bifunctional Catalyst Blend Optimal Control Problem (Problem No. 3 in Table 1)

Luus [6, 7] studied a hard multimodal optimal control problem which has many local optima. As many as 300 local optima have been reported by Esposito and Floudas [8]. The proposed problem is a chemical process which converts methylcyclopentane to benzene in a tubular reactor and described by a set of following seven differential equations:

$$\dot{x}_1 = -k_1 x_1,\tag{4}$$

$$\dot{x}_2 = k_1 x_1 - (k_2 + k_3) x_2 + k_4 x_5, \tag{8}$$

$$\dot{x}_3 = k_2 x_2,\tag{9}$$

$$\dot{x}_4 = -k_6 x_4 + k_5 x_5,\tag{10}$$

$$\dot{x}_5 = -k_3 x_2 + k_6 x_4 - (k_4 + k_5 + k_8 + k_9) x_5 + k_7 x_6 + k_{10} x_7, \tag{11}$$

$$\dot{x}_6 = k_8 x_5 - k_7 x_6, \tag{12}$$

$$\dot{x}_7 = k_9 x_5 - k_{10} x_7 \tag{13}$$

where  $x_i$ , i = 1, 2, ..., 7 are the mole fractions of the chemical species, and the rate constants  $(k_i)$  are cubic functions of the catalyst blend u(t):

$$k_i = c_{i1} + c_{i2}u + c_{i3}u^2 + c_{i4}u^3, \quad i = 1, 2, ..., 10$$
 (14)

Where the values of the coefficients  $c_{ij}$  are experimentally evaluated are given in Table 1[7]. The mass fraction of the hydrogenation catalyst is bounded as:  $0.6u \le (t) \le 0.9$  and initially  $x[0] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ . This chemical process is operated in steady state and hence  $x_1$  appears at the beginning of the process and  $x_7$  at the end, so  $x_1$  through  $x_7$  can be considered to be placed along the length of the tubular reactor. The objective is to

determine the catalyst blend u along the length of the reactor at time interval  $0 \le t \le t_f$  where  $t_f = 2000$  gm/mol such that the performance index

$$J = x_7(t_c) \times 10^3 \tag{15}$$

representing the benzene concentration at the exit of the reactor, is maximized.

**Table: 1:** value of C

2.918487e-003	-8.045787e-003	6.749947e-003	-1.416647e-003
9.509977e+000	-3.500994e+001	4.283329e+001	-1.733333e+001
2.682093e+001	-9.556079e+001	1.130398e+002	-4.429997e+001
2.087241e+002	-7.198052e+002	8.277466e+002	-3.166655e+002
1.350005e+000	-6.850027e+000	1.216671e+001	-6.666689e+000
1.921995e-002	-7.945320e-002	1.105660e-001	-5.033333e-002
1.323596e-001	-4.692550e-001	5.539323e-001	-2.166664e-001
7.339981e+000	-2.527328e+001	2.993329e+001	-1.199999e+001
-3.950534e-001	1.679353e+000	-1.777829e+000	4.974987e-001
-2.504665e-005	1.005854e-002	-1.986696e-002	9.833470e-003

### 4. Optimal Control of a Non-Linear Stirred Tank Reactor (Problem No. 4 in Table 1)

A first-order irreversible chemical reaction carried out in a continuous stirred tank reactor (CSTR) is a multimodal optimal control problem. It has been tried to solve by stochastic global optimization algorithm by Ali *et al.* [10]. Luus *et al.* [7] tried to solve it by Iterative Dynamic Programming algorithm. This problem is a benchmark optimization problem in Handbook of Test Problems in Local and Global Optimization [9]. This chemical process is modeled by two non-linear differential equations:

$$\dot{x}_1 = -(2+u)(x_1+0.25) + (x_2+0.5) \exp\left(\frac{25x_1}{x_1+2}\right),\tag{16}$$

$$\dot{x}_2 = 0.5 - x_2 - (x_2 + 0.5) \exp\left(\frac{25x_1}{x_1 + 2}\right),\tag{17}$$

u(t) = Flow rate of the cooling fluid,  $x_1$  = Dimensionless steady state temperature and  $x_2$  = Deviation from dimensionless steady state concentration. The optimization objective is to determine suitable value of u so that the performance index

$$J = \int_0^{t_f = 0.72} (x_1^2 + x_2^2 + 0.1 \cdot u^2) dt, \tag{18}$$

is minimizes where the initial conditions are  $x(0) = \begin{bmatrix} 0.0.9 & 0.09 \end{bmatrix}^T$ . Though the problem is unconstrained the initial guess for u(t) lies in  $\begin{bmatrix} 0.0 & 5.0 \end{bmatrix}$ . The integration involved in evaluation is performed using sub-function *ode45* available in MATLAB with relative tolerance set to  $1 \times 10^{-1}$ .

### 5. Tersoff Potential Function Minimization Problem (Problem No. 5 & 6 in Table 1)

Evaluation of inter atomic potentials for covalent systems, particularly for Silicon has been receiving great interest from the researchers. One such potential is Tersoff potential, which governs the interaction of silicon atoms with a strong covalent bonding. Tersoff has given two parameterization of silicon and these are called Sc(B) and Sc(C). If we define the positions of the molecular clusters of N atoms by:

$$\vec{X} = \{\vec{X}_1, \vec{X}_2, ..., \vec{X}_N\},\tag{19}$$

where  $\vec{X}_i s$   $i \in \{1, 2, ..., N\}$  is a three-dimensional vector denoting the coordinates of the  $i_{th}$  atom in Cartesian coordinate system then the total Tersoff potential energy function of the system is a function of atomic co-ordinates and defined as

$$f(\vec{X}_1, ..., \vec{X}_3) = E_1(\vec{X}_1, ..., \vec{X}_3) + ... + E_N(\vec{X}_1, ..., \vec{X}_3)$$
 (20)

Total potential is sum of individual potentials of atoms given by eqn. (21), which further needs calculation of eqn. (22) through (27). The potential of any atom depends on the physical position of their neighbor atoms. Different atoms have different distances and angles

subtended with respect to the other atoms so every atom has deferent energies. Now the Tersoff potential [11] of individual atoms can be formally defined as

$$E_{i} = \frac{1}{2} \sum_{j \neq i} f_{c}\left(r_{ij}\right) \left(V_{R}\left(r_{ij}\right) - B_{ij}V_{R}\left(r_{ij}\right)\right), \quad \forall i$$
(21)

where  $r_{ij}$  is the distance between atoms i and j,  $V_R$  is a repulsive term,  $V_A$  is an attractive term,  $f_c(r_{ij})$  is a switching function and  $B_{ij}$  is a many-body term that depends on the positions of atoms i and j and the neighbors of atom i. These quantities are detailed in [11].

$$B_{ij} = \left(1 + \gamma^{n_1} \xi_{ij}^{n_1}\right)^{-1/2n_1} \tag{22}$$

where  $n_i$  and  $\gamma$  are known fitted parameters [11]. The term  $\xi$ ij for atoms i and j (i.e., for bond ij) is given by:

$$\xi_{ij} = \sum_{k \neq i} f_c \left( r_{ij} \right) g \left( \theta_{ijk} \right) \exp \left( \lambda_3^3 \left( r_{ij} - r_{ik} \right)^3 \right)$$
 (23)

Here  $\xi_{ij}$  describes the contribution of the neighbors of the atom i,  $\xi_{ij}$  increases as the number of k atoms increases but the term  $B_{ij}$  decreases as  $\xi_{ij}$  increases. The exponential term in (23) is designed to reduce the contribution of bonds with length greater than  $\gamma_{ij}$ , so that the distant neighbors of i have a reduced contribution to the bond order term. The term  $\theta_{ijk}$  is the bond angle between bonds ij and ik, and the function g is given by:

$$g(\theta_{ijk}) = 1 + c^2/d^2 - c^2/(d^2 + (h - \cos(\theta_{ijk}))^2)$$
 (24)

The parameter h is the cosine of the optimum bond angle and c and d control the influence of bond angles on the many-body term. The quantities  $\lambda_3$ , c, d and h which appear

in (23) and (24) are also known fitted parameters. The terms  $V_R(r_{ij})$  and  $V_A(r_{ij})$  are given by:

$$V_R\left(r_{ij}\right) = Ae^{-\lambda_l r_{ij}} \tag{25}$$

$$V_A(r_{ij}) = Be^{-\lambda_2 r_{ij}} \tag{26}$$

where  $A,B,\ \lambda_1$  and  $\lambda_2$  are given fitted parameters. The switching function  $f_c(r_{ij})$  restricts the potential calculations to nearest neighbors only and ensures that atomic interactions decay smoothly to zero as the separation distance increases. It is given by

$$f_{c}(r_{ij}) = \begin{cases} 1, & \text{if } r_{ij} \leq R - D \\ \frac{1}{2} - \frac{1}{2} \sin \left[ \pi \left( r_{ij} - R \right) / D \right], & \text{if } R - D \langle r_{ij} \langle R + D \rangle \\ 0, & \text{if } r_{ij} \geq R + D \end{cases}$$

$$(27)$$

The two sets of parameter values respectively for Si(B) and Si(C) is tabulated in Table 2 [12]. Though Tersoff potential problem is a  $N \times 3$  dimensional in 3-dimentional space, the no. of dimensions to be evaluated can be decreased in light of the fact that it depends on relative position of atom instead of actual Cartesian coordinates. One atom can be permanently put on the origin and second atom on the positive x-axis. Thus for a 3-atom system the actual no. of variables is four instead of nine. Now, for each additional atom added to the system, the no. of variables increases by three (three Cartesian coordinates of an additional atom). Thus, in general, for the system of N atoms, the no. of unknown variables is  $n = 3 \times N - 6$ . Now the cluster X of N atoms can be redefined as

$$x = \{x_1, x_2, ..., x_n\}, x \in IR^{(3N-6)}$$
(28)

The search region for both Si (B) and Si(C) model of Tersoff potential is

$$\Omega = \left\{ \left\{ x_1, x_2, ..., x_n \right\} \mid -4.25 \le x_1, x_i \le 4.25, i = 4, ..., n0 \le x_2 \le 4, 0 \le x_3 \le \pi \right\}$$
 (29)

The potential (29) is a complex and differentiable function whose partial derivatives can be found in [11]. The cost function now can be redefined as:

$$f(x) = E_1(x) + E_2(x) + \dots + E_N(x), \quad x \in \Omega$$
(30)

Note that in this case also the variables are initialized in the same way as was done for the Lennard-Jones potential minimization problem.

### 6. Spread Spectrum Radar Polly phase Code Design (Problem No. 7 in Table 1)

When designing a radar-system that uses pulse compression, great attention must be given to the choice of the appropriate waveform. Many methods of radar pulse modulation that make pulse compression possible are known. Polyphase codes are attractive as they other lower side-lobes in the compressed signal and easier implementation of digital processing techniques. Later Dukic and Do-brosavljevic [13] proposed a new method for polyphase pulse compression code synthesis, which is based on the properties of the aperiodic autocorrelation function and the assumption of coherent radar pulse processing in the receiver. The problem under consideration is modeled as a min–max nonlinear non-convex optimization problem in continuous variables and with numerous local optima. It can be expressed as follows:

global 
$$\min_{x \in X} f(x) = \max\{\phi_1(x), ..., \phi_{2m}(x)\},$$
 (31)

$$X = \{(x_1, ..., x_n) \in \mathbb{R}^n \mid 0 \le x_j \le 2\pi, j = 1, ..., n\},\$$

where m = 2n - 1 and

$$\phi_{2i-1}(x) = \sum_{j=i}^{n} \cos\left(\sum_{k=|2i-j-1|+1}^{j} x_k\right), \quad i = 1, ..., n$$
(32)

$$\phi_{2i}(x) = 0.5 + \sum_{j=i+1}^{n} \cos\left(\sum_{k=|2i-j|+1}^{j} x_k\right), \quad i = 1, ..., n-1$$
(33)

$$\phi_{m+i}(x) = -\phi_i(x), \quad i = 1,...,m$$
 (34)

Here the objective is to minimize the module of the biggest among the samples of the socalled auto-correlation function which is related to the complex envelope of the compressed radar pulse at the optimal receiver output, while the variables represent symmetrized phase differences. The problem is NP-hard and previously a few approaches to solve it by using GAs and some variants of DE (Differential Evolution) can be found in [14, 15]. This problem belongs to the class of continuous min–max global optimization problems. They are characterized by the fact that the objective function is piecewise smooth.

## 7. Transmission Network Expansion Planning (TNEP) Problem: (Problem No. 8 in Table 1)

### **TNEP** without security constraints:

The simple TNEP [16, 17] without security constraints determines the set of new lines to be constructed such that the cost of expansion plan is minimum and no overloads are produced during the planning horizon. A DC power flow based model is used for TNEP. The TNEP without security constraints, can be stated as follows,

$$\min v = \sum_{l \in \Omega} c_l n_l \tag{35}$$

s.t.

$$S f + g = d (36)$$

$$f_{l} - \gamma_{l} (n_{l}^{0} + n_{l}) (\Delta \theta_{l}) = 0, \text{ for } l \in 1, 2, \dots,$$
 (37)

$$\left| f_{l} \right| \le \left( n_{l}^{0} + n_{l} \right) \overline{f_{l}}$$
, for  $l \in 1, 2, \dots, nl$  (38)

$$0 \le n_l \le \overline{n_l} \tag{39}$$

 $f_i$  and  $\theta_i$  are unbounded,

 $n_l \ge 0$ , and integer, for  $l \in 1,2,\ldots,nl$ ,

 $l \in \Omega$ .

 $c_l$ : cost of line added in  $l^{th}$  right-of-way,

S: branch-node incidence transposed matrix of the power system,

f: vector with elements  $f_{i}$ ,

 $\gamma_l$ : susceptance of the circuit that can be added to  $l^{th}$  right-of-way,

 $n_i$ : the number of circuits added in  $l^{th}$  right-of-way,

 $n_i^0$ : no. of circuits in the base case,

 $\Delta\theta$ : phase angle difference in  $l^{th}$  right-of way,

 $f_l$ : total real power flow by the circuit in  $l^{th}$  right-of-way,

 $\overline{f_l}$ : maximum allowed real power flow in the circuit in  $l^{th}$  right-of-way,

 $\overline{n_i}$ : maximum number of circuits that can be added in  $l^{th}$  right-of-way,

 $\Omega$ : set of all right-of-ways,

nl: total number of lines in the circuit.

The objective is to minimize the total investment cost of the new transmission lines to be constructed, satisfying the constraint on real power flow in the lines of the network. Constraint (38) represents the power balance at each node. Constraint (37) is the real power flow equations in DC network. Constraint (38) represents the line real power flow constraint. Constraint (39) represents the restriction on the construction of lines per corridor (R.O.W). The transmission lines added in any right-of-way are the decision variables.

The cost of each solution to the TNEP without security constraints can be obtained using the eqn.(40) as shown below:

Minimize:

$$f = \sum_{l \in \Omega} c_l n_l + W_1 \sum_{ol} (abs (f_l) - \overline{f_l}) + W_2 (n_l - \overline{n_l})$$
(40)

ol: represents the set of overloaded lines.

The objective of the TNEP is to find the set of transmission lines to be constructed such that the cost of expansion plan is minimum and no overloads are produced during the planning horizon. Hence, first term in the equation (40) indicates the total investment cost of a transmission expansion plan. The second term is added to the objective function for the real power flow constraint violations. The third term is added to the objective function if maximum number of circuits that can be added in  $l^{th}$  right-of-way exceeds the maximum limit.  $W_1$ ,  $W_2$  are constants. The second and third terms are added to the fitness function only in case of violations.

### 8. Large Scale Transmission Pricing Problem (Problem No. 9 in Table 1)

In the modern era of deregulated power systems, transmission pricing is one of the extensively debated issues in literature [18-20]. The unbundling of vertically integrated utilities creates transmission owner as a separate identity. The transmission pricing, as per generic perception, tackles the issue of allocating the fixed costs of transmission to various stake-holders. Many factors influence the decision about the scheme of transmission pricing to be adopted. Some of the factors could be: ex-ante / ex-post pricing, market based / non-market methods, methods for centralized or decentralized markets, whether loss is part of transmission pricing or not, etc. The list is quite exhaustive and hence is the number of proposed methods of transmission pricing.

Equivalent Bilateral Exchange (EBE) is one such method introduced in [19] that works on the linearized model of the system. The original scheme is developed for a pool market (non-transaction based), in order to calculate the final transmission charges for each node. The method imposes a rule on the established power flow snap-shot. The rule is based on the assumption that every generator contributes to every load. The amount of contribution is decided in proportionate manner. The method provides fair price signals and proves to be useful in pool system, where, bilateral transactions are non-existent.

The original EBE method [19] creates a load-generation interaction matrix (equivalent bilateral exchange) based on the following proportionality principle:

$$GD_{ij} = \frac{P_{Gi}P_{Dj}}{P_D^{sys}},\tag{41}$$

where,  $GD_{ij}$  is the amount of equivalent bilateral exchange that takes place between generator i and load j and  $P_D^{sys}$  is the total load in the system. Now the fraction of power flow in line k due to bilateral exchange between generation at bus i and demand at bus j is evaluated for all equivalent bilateral power exchanges using DC load flow or by using PTDF [18]. The net flow in the line k is the absolute sum of all the flows due to all the transactions. The net flow in the line can be expressed in terms of equivalent bilateral power exchanges as

$$pf_k = \sum_{i} \sum_{j} \left| \gamma_{ij}^k \right| GD_{ij} \tag{42}$$

Equation (43) shows that creating an equivalent bilateral transaction matrix using proportionality principle is not the only way to do the pricing. There are multiple solutions possible. Though proportionate principle is logical or intuitive way of decomposing, it needs to be seen how the multiplicity of solutions can be used for proper applications.

### Problem Statement

If there are pre-existing bilateral transactions, it creates a division amongst the existing customers: Bilateral customers and pool customers. Hence, we suggest a different transmission pricing schemes for these two sets of customers. Since the bilateral transactions are a-priori known, the usage rate for the BT are calculated using PTDF and evaluating usage of lines by BTs. Thus, in step 1, some of the total charges to be recovered are attributed to BTs. The rest of the charges are now recovered through pool customers. However, the allocation of these remaining charges and calculation of rates thereby, is done using optimization. As mentioned earlier, the optimization helps in exploring the multiple solutions in deciding the equivalent bilateral exchanges such that the charges of pool customers are close to those had BTs been absent. The formulation is discussed next.

### Problem Formulation

An input file, containing the bus specifications, line specifications, fixed cost to be recovered and bilateral transactions data, is considered as the problem definition. The sensitivity of a line connecting bus m and n for a transaction between buses i and j are given by [18] as

$$\gamma_{ij,mn} = \frac{X_{mi} - X_{ni} - X_{mj} - X_{nj}}{x_{mn}},$$
(43)

where  $x_{mn}$  is the reactance of line connecting buses m and n;  $X_{mi}$ ,  $X_{ni}$ ,  $X_{mj}$  and  $X_{nj}$  entries of the X matrix[18].

Usage rates with pool only market:

The EBE method [19] is applied on the given data and the transmission charges in Rs/Mwhr are evaluated at each bus. Let  $R_{ge}^{i}$  is the obtained transmission usage rate for a generation at bus i and  $R_{de}^{j}$  be the obtained transmission usage rate for a demand at bus j.

Usage rates with pool and bilateral transactions:

i) The equivalent bilateral transactions between pool generations and pool demands are obtained by minimizing usage rate deviations due to bilateral transactions i.e.

$$\frac{\min_{GD_{ij}} F(GD_{ij}) = \left[ \sum_{j} GD_{ij} \left[ \sum_{k} \gamma_{ij}^{k} \left[ \sum_{j} \sum_{j} \left| GD_{ij} \cdot \gamma_{ij}^{k} \right| + \sum_{i} \sum_{j} \left| BT_{ij} \cdot \gamma_{ij}^{k} \right| \right] \right] \right] - R_{ge}^{i} + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] \right] + \sum_{i} \left[ \sum_{j} \left| P_{gi} - P_{gi}^{i} \right| + \sum_{i} \left| P_{gi} - P_{gi}^{i} \right| \right] \right] + \sum_{i} \left[ P_{gi} - P_{gi}^{i} \right] \right] + \sum_{i} \left[ P_{gi} - P_{gi}^{i} \right] \right] + \sum_{i} \left[ P_{gi} - P_{gi}^{i} \right]$$

$$+\sum_{j}\left[\frac{\sum_{i}GD_{ij}\left[\sum_{k}\gamma_{ij}^{k}\left[\frac{FC^{k}}{\left[\sum_{i}\sum_{j}\left|GD_{ij},\gamma_{ij}^{k}\right|+\sum_{i}\sum_{j}\left|BT_{ij},\gamma_{ij}^{k}\right|\right]\right]\right]}{P_{dj}-P_{dj}^{'}}-R_{de}^{j}\right]$$

$$(44)$$

Subject to constraints:

$$\sum_{i} GD_{ij} = P_{gi} - P_{gi}^{i} \quad \forall i \quad , \tag{45}$$

$$\sum_{j} GD_{ij} = P_{gi} - P_{gi}^{i} \quad \forall i \quad ,$$

$$\sum_{i} GD_{ij} = P_{dj} - P_{dj}^{i} \quad \forall j \quad ,$$

$$(45)$$

where,  $GD_{ij}$  is the equivalent bilateral transaction that need to be evaluated;  $FC^k$  is the fixed cost of a line k that needs to be recovered in Rs/hr;  $BT_{ij}$  stands for bilateral transaction between generator at bus i and demand at bus j;  $P_{gi}$  is total generation at bus i;  $P_{gi}$  sum of generations due to all bilateral transactions;  $P_{dj}$  total demand at bus j;  $P_{dj}$  sum of demands due to all bilateral transactions at bus j.

### ii) Charges

The line usage charge in Rs/MW hr for a line k is given by

$$r^{k} = \frac{FC^{k}}{\left[\sum_{i}\sum_{j}\left|GD_{ij}.\gamma_{ij}^{k}\right| + \sum_{i}\sum_{j}\left|BT_{ij}.\gamma_{ij}^{k}\right|\right]}$$
(47)

The usage rate for pool generation at bus i is given by

$$URPG^{i} = \frac{\left[\sum_{j} GD_{ij} \left(\sum_{k} \left(\gamma_{ij}^{k}, r^{k}\right)\right)\right]}{P_{gi} - P_{gi}^{'}} . \tag{48}$$

The usage rate for bilateral transacted generation at bus i is given by

$$URBG^{i} = \frac{\left[\sum_{j} BT_{ij} \left(\sum_{k} \left(\gamma_{ij}^{k} r^{k}\right)\right)\right]}{P_{oi}^{'}} . \tag{49}$$

The usage rate for pool demand at bus *j* is given by

$$URPD^{j} = \frac{\left[\sum_{i} GD_{ij} \left(\sum_{k} \left(\gamma_{ij}^{k} r^{k}\right)\right)\right]}{P_{dj} - P_{dj}^{'}} . \tag{50}$$

The usage rate for bilateral transacted demand at bus *j* is given by,

$$URBD^{j} = \frac{\sum_{j} BT_{ij} \left( \sum_{k} \gamma_{ij}^{k} r^{k} \right)}{P_{di}}$$
(51)

The present instantiation of the problem is on IEEE 30 bus system. We have assumed that the fixed cost to be recovered is US\$ 100/hr and the cost of each element is proportional to the line reactance as the element costs are unavailable for this network system. First, a base case DC load flow is run and the charges for each bus using original EBE scheme are calculated. Then, we superimpose additional bilateral transactions over the base case data to realize a combined pool and bilateral market. The charges for BTs and Pool customers can be calculated using equations (48 - 51). The charges for generators and loads in Pool scheme are calculated using equation (48) and (50) respectively, such that the objective function of equation (44) is minimized.

### 9. Circular Antenna Array Design Problem (Problem No. 10 in Table 1)

Circular shaped antenna arrays find various applications in sonar, radar, mobile and commercial satellite communication systems [21 - 23]. Let us consider N antenna elements spaced on a circle of radius r in the x-y plane. This is shown in figure 1 and the antenna elements are said to constitute a circular antenna array. The array factor for the circular array is written as follows,

$$AF(\phi) = \sum_{n=1}^{N} I_n \exp\left[jkr\left(\cos\left(\phi - \phi_{ang}^n\right) - \cos\left(\phi_0 - \phi_{ang}^n\right)\right) + \beta_n\right]$$
 (52)

where,

 $\phi_{ang}^{n} = 2\pi(n-1)/N$  is the angular position of the  $n^{th}$  element on the x-y plane,

Kr = Nd where k is the wave-number, d is the angular spacing between elements and r is the radius of the circle defined by the antenna array,

 $\phi_0$  is the direction of maximum radiation,

 $\phi$  is the angle of incidence of the plane wave,  $I_n$  is the current excitation and  $\beta_n$  is the phase excitation of the  $n^{th}$  element.

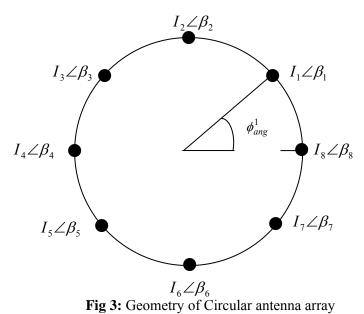
Here we shall vary the current and phase excitations of the antenna elements and try to suppress side-lobes, minimize beamwidth and achieve null control at desired directions. We consider a symmetrical excitation of the circular antenna array i. e. the relations given below will hold,

$$I_{n/2+1} \angle \beta_{n/2+1} = conj(I_1 \angle \beta_1),$$

$$I_{n/2+2} \angle \beta_{n/2+2} = conj(I_1 \angle \beta_1),....$$

$$I_n \angle \beta_n = conj(I_{n/2} \angle \beta_{n/2})$$

The objective function is taken as,



$$OF = \left| AR\left(\varphi_{sll}, \vec{I}, \vec{\beta}, \varphi_{0}\right) \right| / \left| AR\left(\varphi_{\max}, \vec{I}, \vec{\beta}, \varphi_{0}\right) \right| + 1 / DIR\left(\varphi_{0}, \vec{I}, \vec{\beta}\right) + \left| \varphi_{0} - \varphi_{des} \right| + \sum_{k=1}^{mim} \left| AR\left(\varphi_{k}, \vec{I}, \vec{\beta}, \varphi_{0}\right) \right|$$

(53)

The first component attempts to suppress the sidelobes.  $\phi_{sll}$  is the angle at which maximum sidelobe level is attained. The second component attempts to maximize directivity of the array pattern. Nowadays directivity has become a very useful figure of merit for comparing array patterns. The third component strives to drive the maxima of the array pattern close to the desired maxima  $\phi_{des}$ . The fourth component penalizes the objective function if sufficient null control is not achieved. num is the number of null control directions and  $\phi_k$  specifies the  $k^{th}$  null control direction.

### Below we provide the instantiation of the design problem for this competition:

```
Number of elements in circular array = 12 x1= Any string within bounds null= [50,120] in radians (no null control) phi_desired= 180^\circ distance= 0.5
```

Here x1 denotes the input string and the readers may look at the *read\_me.txt* file in the folder Prob\_9\_Circ\_Antenna for more details.

### 10. Dynamic Economic Dispatch (DED) Problem (Problem No. 11.1 and 11.2 in Table 1)

The Dynamic Economic Dispatch (DED) problem follows the charecteristics of the hourly dispatch problem, but here the power demand varies with each hour and the power generation schedule for 24 hours is to be determined. We can say that the dimension of the DED problem is 24 times that of the static ELD problem.

### **Objective Function**

The objective function corresponding to the production cost can be approximated to be a quadratic function of the active power outputs from the generating units. Symbolically, it is represented as:

Minimize: 
$$F_c = \sum_{k=1}^{T} \sum_{i=1}^{N_G} F_{ih} (P_{ih})$$
 (54)

where 
$$F_{it}(P_{it}) = a_i P_{it}^2 + b_i P_{it} + c_i$$
,  $i = 1, 2, 3, ..., N_G$ 

is the expression for cost function corresponding to  $i^{th}$  generating unit and  $a_i$ ,  $b_i$  and  $c_i$  are its cost coefficients,  $P_{it}$  is the real power output (in MW) of  $i^{th}$  generator corresponding to time period t,  $N_G$  is the number of online generating units to be dispatched, T is the total time period of dispatch. The cost function for unit with valve point loading effect is calculated by using:

$$F_{it}(P_{it}) = a_i P_{it}^2 + b_i P_{it} + c_i + \left| e_i \sin \left( f_{it} \left( P_{it}^{\min} - P_{it} \right) \right) \right|$$
 (55)

where  $e_i$  and  $f_i$  are the cost coefficients corresponding to valve point loading effect. Due to the valve point loading the solution may be trapped in the local minima and it also increases the nonlinearity in the system. This constrained DEDP problem is subjected to a variety of constraints depending upon assumptions and practical implications. These include power balance constraints to take into account the energy balance; ramp rate limits to incorporate dynamic nature of DEDP problem and prohibited operating zones. These constraints are discussed as under.

### **Power Balance Constraints:**

This constraint is based on the principle of equilibrium between total system generation and total system loads (PD) and losses (PL). That is,

$$\sum_{i=1}^{N_G} P_{it} = P_{Dt} + P_{Lt} \tag{56}$$

where  $P_{Lt}$  is obtained using B- coefficients, given by

$$P_{Lt} = \sum_{i=1}^{N_G} \sum_{j=1}^{N_G} P_{it} B_{ij} P_{jt}$$
(57)

### **Generator Constraints:**

The output power of each generating unit has a lower and upper bound so that it lies in between these bounds. This constraint is represented by a pair of inequality constraints as follows:

$$P_i^{\min} \le P_{it} \le P_i^{\max} \tag{58}$$

where,  $P_i^{\,min}$  and  $P_i^{\,max}$  are lower and upper bounds for power outputs of the  $i^{th}$  generating unit in MW.

### **Ramp Rate Limits:**

One of unpractical assumption that prevailed for simplifying the problem in many of the earlier research is that the adjustments of the power output are instantaneous. However, under practical circumstances ramp rate limit restricts the operating range of all the online units for adjusting the generator operation between two operating periods. The generation may increase or decrease with corresponding upper and downward ramp rate limits. So, units are constrained due to these ramp rate limits as mentioned below.

If power generation increases,  $P_{it} - P_i^{\ t-1} \le UR_i$ 

If power generation decreases,  $P_i^{t-1} - P_{it} \le DR_i$ 

where  $P_i^{t-1}$  is the power generation of unit i at previous hour and  $UR_i$  and  $DR_i$  are the upper and lower ramp rate limits respectively. The inclusion of ramp rate limits modifies the generator operation constraints as follows

$$\max(P_i^{\min}, UR_i - P_i) \le P_i \le \min(P_i^{\max}, P_i^{t-1} - DR_i)$$
 (59)

### **Constraints Handling:**

To evaluate the fitness of each individual in the population in order to minimize the fuel costs while satisfying unit and system constraints, the following fitness-function model is adopted for simulation in this work:

$$f_k = \sum_{t=1}^n \sum_{i=1}^N F_i(P_{it}) + \lambda_1 \left( \sum_{t=1}^n \sum_{i=1}^N P_{it} - P_{Dt} \right)^2 + \lambda_r \left( \sum_{t=1}^n \sum_{i=1}^N P_{it} - P_{r \text{lim}} \right)^2$$
(60)

where  $\lambda_1$  and  $\lambda_r$  are penalty parameters, n is the number of hours, N is the number of units. The penalty factors regulate the objective function such that the algorithm gives higher cost value rather than directly judging the solutions as infeasible. The penalty term reflects the violation of the equality constraint and assigns a high cost of penalty function. The  $P_{rlim}$  is defined by

$$P_{r lim} = \begin{bmatrix} P_{i(t-1)} - DR_{i}, P_{it} < P_{i(t-1)} - DR_{i} \\ P_{i(t-1)} + UR_{i}, P_{it} > P_{i(t-1)} + UR_{i} \\ P_{it}, otherwise \end{bmatrix}$$
(61)

# 11. Static Economic Load Dispatch (ELD) Problem (Problem No. 11.3 – 11.7 in Table 1)

The static ELD problem is about minimizing the fuel cost of generating units for a specific period of operation, usually one hour of operation, so as to accomplish optimal generation dispatch among operating units and in return satisfying the system load demand, generator operation constraints with ramp rate limits and prohibited operating zones. Hereby, two alternative models for ELD are considered viz. one with smooth cost functions and the other with non-smooth cost function as detailed below.

### **Objective Function**

The objective function corresponding to the production cost can be approximated to be a quadratic function of the active power outputs from the generating units. Symbolically, it is represented as:

Minimize: 
$$F = \sum_{i=1}^{N_G} f_i(P_i)$$
 (62)

where  $f_i(P_i) = a_i P_i^2 + b_i P_i + c_i$ ,  $i = 1, 2, 3, ..., N_G$  is the expression for cost function corresponding to  $i^{th}$  generating unit and  $a_i$ ,  $b_i$  and  $c_i$  are its cost coefficients.  $P_i$  is the real power output (in MW) of  $i^{th}$  generator corresponding to time period t.  $N_G$  is the number of online generating units to be dispatched. The cost function for unit with valve point loading effect is calculated by using:

$$f_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \sin \left( f_i \left( P_i^{\min} - P_i \right) \right) \right|$$
(63)

where  $e_i$  and  $f_i$  are the cost coefficients corresponding to valve point loading effect. This constrained ELD problem is subjected to a variety of constraints depending upon assumptions and practical implications. These include power balance constraints to take into account the energy balance; ramp rate limits to incorporate dynamic nature of ELD problem and prohibited operating zones. These constraints are discussed as under.

### **Power Balance Constraints or Demand Constraints**

This constraint is based on the principle of equilibrium between total system generation and total system loads  $(P_D)$  and losses  $(P_L)$ . That is,

$$\sum_{i=1}^{N_G} P_i = P_D + P_L \tag{64}$$

where P<sub>L</sub> is obtained using B- coefficients, given by

$$P_{L} = \sum_{i=1}^{N_{G}} \sum_{j=1}^{N_{G}} P_{i} B_{ij} P_{j} + \sum_{i=1}^{N_{G}} B_{0i} P_{i} + B_{00}$$
(65)

### **Generator Constraints**

The output power of each generating unit has a lower and upper bound so that it lies in between these bounds. This constraint is represented by a pair of inequality constraints as follows:

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{66}$$

where,  $P_i^{min}$  and  $P_i^{max}$  are lower and upper bounds for power outputs of the  $i^{th}$  generating unit.

### **Ramp Rate Limits**

One of unpractical assumption that prevailed for simplifying the problem in many of the earlier research is that the adjustments of the power outputs are unbounded. However, under practical circumstances ramp rate limit restricts the operating range of all the online units for adjusting the generator operation between two operating periods. The generation may increase or decrease with corresponding upper and downward ramp rate limits. So, units are constrained due to these ramp rate limits as mentioned below:

If power generation increases,  $P_i - P_i^{t-1} \le UR_i$ 

If power generation decreases,  $P_i^{t-1} - P_i \le DR_i$ 

where  $P_i^{t-1}$  is the power generation of unit i at previous hour and  $UR_i$  and  $DR_i$  are the upper and lower ramp rate limits respectively. The inclusion of ramp rate limits modifies the generator operation constraints as follows:

$$\max(P_i^{\min}, UR_i - P_i) \le P_i \le \min(P_i^{\max}, P_i^{t-1} - DR_i)$$

### **Prohibited Operating Zones**

The generating units may have certain zones where operation is restricted on the grounds of physical limitations of machine components or instability e.g. due to steam valve or vibration in shaft bearings. Consequently, discontinuities are produced in cost curves corresponding to

the prohibited operating zones. So, there is a quest to avoid operation in these zones in order to economize the production. Symbolically, for a generating unit i,

$$P_i \le \widetilde{P}^{pz}$$
 and  $P_i \ge \widehat{P}^{pz}$ 

where  $\check{P}^{pz}$  and  $\widehat{P}^{pz}$  are the lower and upper limits of a given prohibited zone for generating unit i.

### **Conflicts in Constraints Handling**

If ever, the maximum or minimum limits of generation of a unit as given lie in the prohibited zone for that generator, then some modifications are to be made in the upper and lower limits for the generator constraints in order to avoid the conflicts. In case, maximum limit for a generator lies in the prohibited zone, the lower limit of the prohibited zone is taken as the maximum limit of power generation for that particular generator. Similarly, care is taken in case the minimum limit of power generation of a generator lies in the prohibited zone by taking upper limit of the prohibited zone as the lower limit of power generation for that generator.

### 12. Hydrothermal Scheduling Problem (Problem No. 11.8 – 11.10 in Table 1)

Hydrothermal scheduling can be either a short term or a long term problem. Short term refers to duration of typically 24 hours, while long term refers to duration of weeks or months. The primary intent of short-term scheduling of a hydrothermal power system is to schedule the power generations of the thermal and hydro units in the system to satiate the load demands, in the scheduling duration of one day or a few days, in accordance with the various constraints put on the hydraulic systems and the power system networks. Generally, the objective function to be minimized in a hydrothermal scheduling problem is the overall fuel cost of thermal units for the given short term. The hydrothermal system considered here is extremely complex and involves nonlinear relationships of the decision variables, cascaded nature of hydraulic network, water carry delays and time link between the consecutive schedules, that

make the problem of discovering global optimum cost difficult using regular optimization methods.

The system considered comprises of a multi-chain cascaded network of four hydro plants and an equivalent thermal power plant. If there are multiple thermal units, they can be taken together and considered as an equivalent thermal network. The distribution of power among those thermal units can be solved as a different and independent problem. Our objective here would be to maximize the output of the hydro units such that thermal unit takes up only minimum load. Considering the fact that we have to schedule the water discharges of four hydro units for 24 hours, the dimension of the problem is 96. Such huge dimension can take conventional methods years to solve using even a computer.

### **Objective Function**

The total fuel cost for operation of the thermal system so as to meet the load demands in the scheduling period is given by F. The objective function is formulated as

$$Minimize F = \sum_{i=1}^{M} f_i(P_{Ti})$$
(67)

where  $f_i$  is the cost function corresponding to the equivalent thermal unit's power generation  $P_{Ti}$  at  $i^{th}$  interval. M is the total number of intervals considered for the short term schedule. The cost function  $f_i$  can be written as:

$$f_i(P_{Ti}) = a_i P_{Ti}^2 + b_i P_{Ti} + c_i + \left| e_i \sin \left( f_i \left( P_{Ti}^{\min} - P_{Ti} \right) \right) \right|$$
(68)

The minimization problem is subject to various system constraints.

### **Demand constraints**

This constraint comes from the principle of energy conservation. The total power produced by thermal unit and hydro units put together should be satiating both the power demand and the power loss occurring.

$$P_{T(i)} + \sum_{k=1}^{N} P_{H(k,i)} = P_{D(i)} + P_{Loss(i)}$$
(69)

where  $P_{Hki}$  represents the power generated by the  $k^{th}$  hydro unit at the  $i^{th}$  interval;  $P_{Di}$  and  $P_{Lossi}$  represents the power demand and power loss at the  $i^{th}$  interval respectively. N is the total number of hydro units.

### Thermal generator constraints

The equivalent thermal generator can generate power only between a certain lower and upper limits at any i<sup>th</sup> interval.

$$P_{T}^{\min} \leq P_{T(i)} \leq P_{T}^{\max}$$

### Hydro generator constraint

Each of the hydro plant's power generation must belong to its upper and lower bounds of operation.

$$P_{H(k)}^{\,\text{min}} \leq P_{H(k,\,i)} \leq P_{H(k)}^{\,\text{max}}$$

### Reservoir capacity constraint

The volume of each reservoir's storages at any i<sup>th</sup> interval must lie in between the lowest and highest capacity limits of the reservoir.

$$V_{(k)}^{\,\text{min}}\, \leq V_{(k,\,i)} \leq V_{(k)}^{\,\text{max}}$$

Also, the reservoirs have restrictions on the initial and final storage volume they can possess.

$$V_{\scriptscriptstyle (k,0)} = V_{\scriptscriptstyle (k)}^{\scriptscriptstyle initial} \quad \quad V_{\scriptscriptstyle (k,M)} = V_{\scriptscriptstyle (k)}^{\scriptscriptstyle final}$$
 and

### Water discharge constraint

The water discharge rate of each reservoir must belong to its minimum and maximum operating limits at all intervals.

$$Q_{(k)}^{\ min} \ \leq \ Q_{(k,\ i)} \ \leq \ Q_{(k)}^{\ max}$$

### Hydraulic continuity constraint

The volume stored in the  $k^{th}$  reservoir for the  $(i+1)^{th}$  interval is found from the following continuity equation.

$$V_{(k, i+1)} = V_{(k, i)} + \sum_{j=\Omega_{(k)}} (Q_{(j, i-\tau)} + S_{(j, i-\tau)}) - Q_{(k, i)} - S_{(k, i)} + R_{(k, i)}$$
(70)

where  $\Omega_{(k)}$  is the index set of the upstream reservoirs contributing to the  $k^{th}$  reservoir,  $\tau$  is the time delay occurring for the water in  $j^{th}$  upstream reservoir to reach the  $k^{th}$  reservoir. S and R represent the spillage and inflow rate respectively.

### Hydro power generation equation

The hydro power generated by the  $k^{th}$  unit at  $i^{th}$  interval is taken as a function of discharge rate and storage volume of that unit in that interval.

$$P_{H(k, i)} = c_{(1, k)} V_{(k, i)}^{2} + c_{(2, k)} Q_{(k, i)}^{2} + c_{(3, k)} (V_{(k, i)} Q_{(k, i)}) + c_{(4, k)} V_{(k, i)} + c_{(5, k)} Q_{(k, i)} + c_{(6, k)}$$
(71)

where  $c_{(1,k)}$ ,  $c_{(2,k)}$ ,  $c_{(3,k)}$ ,  $c_{(4,k)}$ ,  $c_{(5,k)}$  and  $c_{(6,k)}$  are the constant coefficients of the system for the  $k^{th}$  reservoir.

# 13. Messenger: Spacecraft Trajectory Optimization Problem (Problem No. 12 in Table 1)

A good benchmark to test global optimization algorithms in Space Mission Design related problems is the Multiple Gravity Assist (MGA) problem. In mathematical terms this is a finite dimension global optimisation problem with non linear constraints. It can be used to locate the best possible trajectory that an interplanetary probe equipped with a chemical propulsion engine may take to go from the Earth to another planet or asteroid. The spacecraft is constrained to thrust only at planetary encounters. A detailed description of the problem may be found in [24]. The constraint on the spacecraft thrusting only only at planetary encounters is often unacceptable as it may results in trajectories that are not realistic or that use more propellant than necessary. The MGA-1DSM problem removes most of these limitations. It represents an interplanetary trajectory of a spacecraft equipped with chemical propulsion, able to thrust its engine once at any time during each trajectory leg. Thus the solutions to this problem are suitable to perform preliminary quantitative calculation for real space missions. This comes to the price of having to solve an optimization problem of larger dimensions. The implementation details of this problem are the sum of a number of previously published works [25 - 28].

The "Messenger" trajectory optimization problem represents a rendezvous mission to Mercury modeled as an MGA-1DSM problem. The selected fly-by sequence and other parameters are compatible with the currently flying Messenger mission. With respect to the problem "Messenger" the fly-by sequence is more complex and allows for resonant fly-bys at Mercury to lower the arrival DV. For the twenty-six dimensional global optimization problem, the detailed ranges of each variable, MATLAB codes etc. can be found from the URL:

http://www.esa.int/gsp/ACT/inf/op/globopt/MessengerFull.html

# 14. Cassini 2: Spacecraft Trajectory Optimization Problem (Problem No. 13 in Table 1)

In this problem the objective function (unconstrained) evaluates the DV required to reach Saturn using an Earth – Venus, Venus – Earth, Jupiter - Saturn fly-by sequence with deep space maneuvers. Consider a different model for the Cassini trajectory: deep space maneuvers are allowed between each one of the planets. This leads to a higher dimensional problem with a much higher complexity. We also consider, in the objective function evaluation, a rendezvous problem rather than an orbital insertion as in the MGA model of the Cassini mission. This is the main cause for the higher objective function values reached. For the 12-dimensional state-vector the bounds, the MATLAB code for evaluating the objective functions etc. can be found in the following URL:

http://www.esa.int/gsp/ACT/inf/op/globopt/edvdvdedjds.htm

**Table 1:** Summary of the problems presented

Problem No. (as they	No. of	Constraints	Bounds
should appear in the	Dimensions	Constraints	Dounds
participant papers)	2 mignorous		
participant papers)			
1. Parameter	6	Bound	All dimensions bound between
Estimation for	-	constrained	
Frequency-			[-6.4, 6.35]
Modulated (FM)			
Sound Waves			
Matlab Folder:			
Probs_1_to_8			
2. Lennard-Jones	$3 \times 10 = 30$	Bound	Let whether weighte of the west town which has d
Potential Problem	3~10 - 30	constrained	Let $\vec{x}$ be the variable of the problem, which has three components for three atoms, six components for 4 atoms
1 otentiai i rooteni	(10 atom	Constrained	and so on. The first variable due to the second atom i.e.
Matlab Folder:	problem)		$x_1 \in [0, 4]$ , then the second and third variables are such
	,		
Probs_1_to_8			that $x_2 \in [0,4]$ and $x_3 \in [0,\pi]$ . The coordinates
			$x_i$ for any other atom is taken to be bound in the range:
			$\begin{bmatrix} 1 & i-4 & 1 & i-4 \end{bmatrix}$
			$\left[ -4 - \frac{1}{4} \left  \frac{i-4}{3} \right , 4 + \frac{1}{4} \left  \frac{i-4}{3} \right  \right]$
			where $\lfloor r \rfloor$ is the nearest least integer w.r.t. $r \in \square$ .
3. The Bifunctional	1	Bound	[0.6, 0.9]
Catalyst Blend		constrained	
Optimal Control			
Problem			
1011511			
Matlab Folder:			
Probs_1_to_8			
11005_1_10_6			
4. Optimal Control	1	Unconstrained	No bound, initialization in the range [0, 5]
of a Non-Linear			
Stirred Tank Reactor			
Matlab Folder:			
Drobe 1 to 0			
Probs_1_to_8			

5. Tersoff Potential for model Si (B)  Matlab Folder:  Probs_1_to_8  and  6. Tersoff Potential for model Si (C)  Matlab Folder:  Probs_1_to_8	3×10 = 30 (10 atom problem)	Bound constrained	Let $\vec{x}$ be the variable of the problem which has three components for three atoms, six components for 4 atoms and so on. The first variable due to the second atom i.e. $x_1 \in [0,4]$ , then the second and third variables are such that $x_2 \in [0,4]$ and $x_3 \in [0,\pi]$ . The coordinates $x_i$ for any other atom is taken to be bound in the range: $\left[-4 - \frac{1}{4} \left\lfloor \frac{i-4}{3} \right\rfloor, 4 + \frac{1}{4} \left\lfloor \frac{i-4}{3} \right\rfloor\right]$ where $\lfloor r \rfloor$ is the nearest least integer w.r.t. $r \in \square$ .
7. Spread Spectrum Radar Polly phase Code Design Matlab Folder: Probs_1_to_8	20	Bound constrained	All dimensions bound between $[0,2\pi]$
8. Transmission Network Expansion Planning (TNEP) Problem: Matlab Folder: Probs_1_to_8	7	Equality and inequality constraints	All variables are bounded in the interval [0, 15].
9. Large Scale Transmission Pricing Problem  Matlab Folder:  Prob_9_Transmissio n_Pricing	g*d (g: no. of generator buses, d: no. of load buses) For, IEEE 30 bus system: g=6, d=21	Linear Equality Constraints	$\max\{GD_{ij}\} = \min\{P_{gi} - BT_{ij}, P_{dj} - BT_{ij}\}$ $\min\{GD_{ij}\} = 0$ (plz see the <i>bounds.m</i> file in \Prob_8_Transmission_Pricing folder)
10. Circular Antenna Array Design Problem	12	Bound constrained	First six dimensions in [0.2, 1] and next six dimensions [-180, 180]

Matlab Folder:			
Prob_10_Circ_Ante nna			
11. The ELD Problems			
Matlab Folder:			
Probs_11_ELD_Pac kage			
11.1 DED instance 1	120	Inequality constraints	$P_{min} = [10,20,30,40,50;];$ $P_{max} = [75,125,175,250,300;];$ Lower_Limit = repmat(Pmin,1,24); Upper_Limit = repmat(Pmax,1,24); (in MATLAB)
11.2 DED instance 2	216  (The objective function takes 240 parameters, but 24 of them are fixed at 55 and need not be optimized)	Inequality constraints	$P_{min} = [150,135,73,60,73,57,20,47,20];$ $P_{max} = [470,460,340,300,243,160,130,120,80];$ Lower_Limit = repmat(Pmin,1,24);
11. 3 ELD Instance 1	6	Inequality constraints	6 unit limits (in format [LB1, UB1; LB2, UB2;]): [100, 500;50, 200;80, 300;50, 150; 50, 200;50, 120;];

11.4 ELD Instance 2	13	Inequality constraints	13 unit limits (in format [LB1, UB1; LB2, UB2;]): [0,680;0,360;0,360;60,180;60,180;60,180;60,180;60,180;40,120;40,120;55,120;55,120;5;
11.5 ELD Instance 3	15	Inequality constraints	15 unit limits (in format [LB1, UB1; LB2, UB2;])  [150,455;150,455;20,130;20,130;150,470; 135,460;135,465;60,300;25,162;25,160; 20,80;20,80;25,85;15,55;15,55;]
11.6 ELD Instance 4	40	Inequality constraints	40 unit limits (in format [LB1, UB1; LB2, UB2;]):  [36,114;36,114;60,120;80,190;47,97;68,140;110,300;135,3 00;135,300;130,300;94,375;94,375;125,500;125,500; 125,500;125,500;220,500;220,500;242,550;242,550;254,55 0;254,550;254,550;254,550;254,550;254,550;10,150;10,150 ;10,150;47,97;60,190;60,190;60,190;90,200;90,200;90,200; 25,110; 25,110;25,110;242,550;];
11.7 ELD Instance 5	140	Inequality constraints	140 unit limits (in format [LB1, UB1; LB2, UB2;]):  [71,119;120,189;125,190;125,190;90,190;90,190;280,490; 280,490;260,496;260,496;260,496;260,496;260,506;260,50 9;260,506;260,505;260,506;260,506;260,505;260,505; 260,505;60,505;260,505;260,505;280,537;280,537;280,549; 280,549;260,501;260,501;260,506;260,506;260,506; 260,506;260,500;260,500;120,241;120,241;423,774; 423,769;3,19;3,28;160,250;160,250;160,250;160,250; 160,250;160,250;160,250;160,250;165,504;165,504; 165,504;165,504;180,471;180,561;103,341;198,617; 100,312;153,471;163,500;95,302;160,511;160,511; 196,490;196,490;196,490;196,490;130,432;130,432; 137,455;137,455;195,541;175,536;175,540;175,538;

			175,540;330,574;160,531;160,531;200,542;56,132; 115,245;115,245;115,245;207,307;207,307;175,345; 175,345;175,345;175,345;360,580;415,645;795,984; 795,978;578,682;615,720;612,718;612,720;758,964; 755,958;750,1007;750,1006;713,1013;718,1020;791,954; 786,952;795,1006;795,1013;795,1021;795,1015;94,203; 94,203;94,203;244,379;244,379;244,379;95,190;95,189; 116,194;175,321;2,19;4,59;15,83;9,53;12,37;10,34; 112,373;4,20;5,38;5,19;50,98;5,10;42,74;42,74; 41,105;17,51;7,19;7,19;26,40;];
11.8 Hydrothermal Scheduling Instance 1	96	Inequality constraints	$Q_{min} = [5 \ 6 \ 10 \ 13]; Q_{max} = [15 \ 15 \ 30 \ 25];$ $Lower\_Limit = repmat(Q_{min}, 1, 24);$ $Upper\_Limit = repmat(Q_{max}, 1, 24);$
11.9 Hydrothermal Scheduling Instance 2	96	Inequality constraints	Same as 11.8
11.10 Hydrothermal Scheduling Instance 3	96	Inequality constraints	Same as 11.8
12. Messenger: Spacecraft Trajectory Optimization Problem  Matlab Folder:  Probs_12_to_13_Pac kage	26	Bound constraints	Please see Table 2
13. Cassini 2: Spacecraft Trajectory Optimization Problem	22	Bound constraints	Please see Table 3

Matlab Folder:		
Probs_12_to_13_Pac		
kage		

 Table 2: Variable ranges for "Messenger" space trajectory optimization problem

State	Variable	LB	UB	Units
x(1)	t0	1900	2300	MJD2000
x(2)	Vinf	2.5	4.05	km/sec
x(3)	u	0	1	n/a
x(4)	v	0	1	n/a
x(5)	T1	100	500	days
x(6)	T2	100	500	days
x(7)	Т3	100	500	days
x(8)	T4	100	500	days
x(9)	T5	100	500	days
x(10)	T6	100	600	days
x(11)	eta1	0.01	0.99	days
x(12)	eta2	0.01	0.99	n/a
x(13)	eta3	0.01	0.99	n/a
x(14)	eta4	0.01	0.99	n/a
x(15)	eta5	0.01	0.99	n/a
x(16)	eta6	0.01	0.99	n/a
x(17)	r_p1	1.1	6	n/a
x(18)	r_p2	1.1	6	n/a
x(19)	r_p3	1.05	6	n/a
x(20)	r_p4	1.05	6	n/a
x(21)	r_p5	1.05	6	n/a
x(22)	b_incl1	-pi	pi	n/a
x(23)	b_incl2	-pi	pi	n/a

x(24)	b_incl3	-pi	pi	n/a
x(25)	b_incl4	-pi	pi	n/a
x(26)	b_incl5	-pi	pi	n/a

 Table 3: Variable ranges for "Cassini 2" space trajectory optimization problem

State	Variable	LB	UB	Units
x(1)	t0	-1000	0	MJD2000
x(2)	Vinf	3	5	km/sec
x(3)	u	0	1	n/a
x(4)	v	0	1	n/a
x(5)	T1	100	400	days
x(6)	T2	100	500	days
x(7)	T3	30	300	days
x(8)	T4	400	1600	days
x(9)	T5	800	2200	days
x(10)	eta1	0.01	0.9	n/a
x(11)	eta2	0.01	0.9	n/a
x(12)	eta3	0.01	0.9	n/a
x(13)	eta4	0.01	0.9	n/a
x(14)	eta5	0.01	0.9	n/a
x(15)	r_p1	1.05	6	n/a
x(16)	r_p2	1.05	6	n/a
x(17)	r_p3	1.15	6.5	n/a
x(18)	r_p4	1.7	291	n/a
x(19)	b_incl1	-pi	pi	rad
x(20)	b_incl2	-pi	pi	rad
x(21)	b_incl3	-pi	pi	rad
x(22)	b_incl4	-pi	pi	rad

### 2. Evaluation Criteria

The authors should report the mean, best, and worst objective function values obtained over 25 independent runs. The authors are asked to report these values after executing their algorithms for 50000, 100000, and 150000 FEs (Function Evaluations). The participants should use uniform random initialization within the prescribed search space for each problem. We discourage participants searching for a distinct set of parameters for each problem/dimension/etc. Please provide details on the following whenever applicable:

- a) All parameters to be adjusted
- b) Corresponding dynamic ranges
- c) Guidelines on how to adjust the parameters
- d) Estimated cost of parameter tuning in terms of number of FEs
- e) Actual parameter values used.

If the algorithm requires encoding, then the encoding scheme should be independent of the specific problems and governed by generic factors such as the search ranges.

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